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The Potential of the Internet for Mathematics Education

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Mathematics Education, Mathematic, Didactic, Learning Theory, Teaching Models, Educational Design, eLearning, Blended Learning, Mobile Learning, Microlearning, collaborative Learning

Abstract:

This paper outlines three prototypical modes of teaching and learning as well as their consequences for the design pf technology supported environments for Mathematics education. I distinguish between transfer of knowledge (mode I), acquisition of knowledge (mode II) and construction of knowledge (mode III). Based on this theoretical framework I will develop the notion of the "competence helix" and show how to use technological tools and Learning Management Systems (LMS) for eLearning and Blended Learning.

As a sample demonstration for the usefulness of this approach I will explore in two case studies the potential of Mobile Learning respectively Microlearning for Mathematics education and the potential of the Internet for large scale collaboration on cutting-edge research in Mathematics.

1 Three prototypical models of education

1.1 To receive, absorb, assimilate knowledge (Learning/Teaching I)

In this model the origin of students' knowledge is based on knowledge possessed by the teacher. Teachers know what students need to learn and it is the teachers' responsibility to transfer this knowledge into the students' minds as easily as possible. The transferred knowledge is abstracted knowledge prepared in a special way (the so-called didactical preparation), so that students are able to capture the content not only fast but also to memorise it on a long term basis.

There are some links and relations of this model with behaviourism, a now out-dated learning theory: The central tenet of behaviourism is that our behaviour is the product of our conditioning. So it claims that not our mental processes determine what we do. Learning is therefore a conditioned reflex that takes place through adaptation, a process in which the students' behaviour (reactions) simply result from an appropriate stimulus. To provide the appropriate stimuli is the main activity of teachers according to this theory. These stimuli have to be supported by adequate feedback to emphasise the correct (=desired by the teacher) mode of behaviour. – But later we will see that even learning theoretical notions and definitions by heart demands a lot of active cognitive processes and is not fully covered by the behaviourist model. (For a more developed argument see Baumgartner, 2004).

Typical examples for receiving and absorbing knowledge are definitions: For instance in Geometry: What is a point, a line, an angle? In Algebra: What is a decimal number, a prime number, a fraction? Other important examples are explanations of concepts like number lines or sets. We call this kind of knowledge factual or conceptual knowledge and it includes knowledge of terminology, knowledge of specific details and elements of a certain subject, knowledge of classifications and categories, of principles and generalizations, of theories, models and structures. Regina Bruder (2014) calls these two types of knowledge – following Franz E. Weinert – differently as "smart knowledge" ("intelligentes Wissen").

Even procedural knowledge can be taught respectively learned with this educational mode. But we have to be aware that there is a strategic difference between procedural knowledge and the two other types of knowledge. Factual and conceptual knowledge are purely theoretical knowledge whereas procedural knowledge has a practical aspect too. To know how things have to be done (= to know the procedure) does not ensure that one is really able to do it. Even if one retorts that conceptual knowledge also has to be applied, I would answer that use of a concept is not inherent in its description whereas the specification of a procedure is their process or execution. – All in all this mode of teaching/learning has especially legitimate usage when it comes to low level, static knowledge. I will call the teaching strategy of transferring knowledge as "Teaching I" and the corresponding activity of the side of the learners "Learning I".

For further elaboration of my main argument it is important to note that the organisational structure of this mode is unidirectional. Knowledge goes from the teacher to the student; the teacher "gives", the student has to "take in", to absorb, to assimilate. Whenever a reaction of the student is required it functions as feedback to see if the knowledge transfer has worked successfully and produced the "correct" behaviour. From a systemic point of view we have two clearly defined systems where one system (the teacher) dominates and controls the other system (the learner).

The typical cognitive processes for pupils and students under "Learning I"-mode are remembering and understanding. We can test the successful process of memorizing by retrieving relevant knowledge from the long-term memory through recognizing and recalling. A prototypical device for this kind of knowledge is the enquiry of terminology and the identification of methods and procedures with a multiple-choice test.

Example 1: Is 3/2 [1] a prime number?; [2] a fractional number?; [3] a decimal number?; [4] a mixed number?

Testing understanding is much more complicated and needs sophisticated strategies: Students have to show that they able not only are to repeat (e.g. to remember knowledge) but to construct meaning from their knowledge. Paraphrasing, summarizing and explaining a solution, giving examples and counter examples, inferring and comparing are suitable strategies to demonstrate understanding.

Example 2: (A x B) x C = A x (B x C). Explain why this is correct?

1.2 To acquire, compile, gather knowledge (Learning/Teaching II)

This second teaching model assumes that learning is an active process, which has to be planned, revised and reflected by the learner. The learner itself is an active entity and it is his/her activity, which supports or even is a necessary condition for the learning process.

To understand the differences between Learning/Teaching I and Learning/Teaching II better we have to refine our arguments. Even the simplest form of knowledge transfer from teacher to learner (Learning/Teaching I) needs some activities by the learner (e.g. attention, listening etc.). The very dumb mode of learning by heart requires already a lot of engagement by the learner (e.g. rehearsal of the material to memorise). So even in the teaching model of receiving knowledge nobody will claim that the learner is not a human being in some kind actively involved in learning. The differences are on a more subtle level: In Learning/Teaching I the teacher is not interested to control or even observe the actual learning activities undertaken by the learner. What counts are just the results whereas in Learning/Teaching II the whole learning process with all its intermediate steps, its difficulties and provisional results are under surveillance by the teacher. In Learning/Teaching I learners essentially get the feedback wrong or true whereas in Learning/Teaching II teachers try to help to overcome wrong assumptions, wrong learning attitudes and to assist in the reflection process in order to aid the student to build up a consistent mental model of the subject domain.

Example 3: A middle school student produces the following two errors:

$$\begin{array}{rr} 500 & 312 \\ \underline{-\ 65} & -\ \underline{243} \\ \mathbf{565} & \mathbf{149} \end{array}$$

Instead of just classifying the answers just as "wrong" the model of Learning/Teaching II tries to detect the reason for the wrong calculations. After an analysis it turns out the student was not careless but followed a wrong rule: 0 – N = N; that is, "if a digit is subtracted from 0, the result is the digit." (Brown & Burton, 1978; Marzano & Kendall, 2007, p. 47)

Teaching II has kinship to cognitivism. The modern and today very likely dominant paradigm of cognitivism emphasizes in contrast to behaviourism an inner processes of the brain seeking to differentiate, investigate and bring these processes into mutual relation. Cognitivism seeks to develop a theoretical model for the processing operations between input and output of the brain, which in this case is not regarded as a black box. In contrast to the behaviouristic approach the brain is not merely regarded as a passive container, but as a "device" with its own processing and information capacity.

With respect to learning the basic paradigm of cognitivism consists of problem solving. In Learning/Teaching II the teacher provides (and controls) a learning environment where learners are able to withdraw, to collect, to gather, to compile etc. the necessary information to solve the presented problem or task. The learner has with certain required actions actively to acquire the necessary knowledge, the teacher observes the knowledge acquisition and tries to facilitate this learning process. In Learning/Teaching

II the teacher is a tutor, a facilitator who watches and examines not only the product, but also the process.

Under these premises the teacher designs a specific learning environment and includes some "observation points" in order to be able to give feedback during the learning process. As there is no chance to look into the heads of learners', we as teachers have to provide a communication structure. In contrast to Learning/Teaching I this communication is based on a dual way channel. Feedback is not only used to judge (wrong or right), but to provide means to help to find the correct solution.

Example 4: What is the numeric value of 6! ? Describe every step of your calculation.

Under Learning/Teaching II a simple multiple-choice test would not provide the necessary information to judge the cognitive procedure. The student has not only to provide an answer but also the solution procedure, e.g.: $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$.

Even if the communication goes into both directions this does not necessarily mean that teachers and learners are on equal terms. In Learning/Teaching II the teacher is a kind of moderator or panel chairman, who directs the discussion. But in contrast to Learning/Teaching I it is a real discussion, the moderator (teacher) considers carefully what the student has to say and as a result changes his/her attitude accordingly.

Model II is especially suited for applying knowledge procedures and analysing problems. Under this model teachers guide students, facilitate their work and cognitive processes. Executing, implementing (apply knowledge = carrying out procedures under given circumstances), differentiating and organizing (analyse problem = dissect material into constituents parts, determine how the different parts relate to each other and understand the overall structure and system) are the central cognitive processes supported adequately by Learning/Teaching II.

Please keep in mind that my description of the different learning/teaching models is conceptual. So the apparently differences between model I and model II could be very small. Concerning Learning/Teaching I it could even happen that there are tasks and problems presented, but just presented. On the one hand there are no built in observation points to facilitate the learning process, but on the other hand in modern curricula nowadays we have permanent test situations meaning that a complex learning process is divided into many small learning products. In our understanding these "observation points" are test situations to judge the learning product. They give learners hints if they are on the right or wrong track, but these check points do not serve as an individual help provided by the teacher. They are just interim judgements. Even if teachers do react (for instance if many students have failed) by providing (e.g. presenting) additional information their teaching mode remains in the boundaries of model Learning/Teaching I.

There is a central difference to check points in Learning/Teaching I compared to Learning/Teaching II. Observation points serve in the first model to improve the transfer of knowledge (more precise, more concise, more effective etc.) to the audience, whereas in the second model the individual learner is supported to progress. To get the required status information from the learner a special learning mood has to be generated. Learners must trust teachers that they do not exploit their bad performance to their disadvantage.

1.3 To develop, to invent, to construct knowledge (Learning/Teaching III)

In the model of Learning/Teaching II teachers present all problems and tasks themselves. This approach has various consequences:

- Only the teacher practices the art of inventing and presenting problems. The student is taught to solve problems but not to "invent" and present them.
- For pedagogical reasons the problems chosen are mostly based on a simplified and rectified dataset and the solutions are often unambiguous.
- For didactical reasons the problems are clearly cut and cleaned up so that the task at hand is evident and the solution is straight forward so that the problem can be solved in the limited time the curriculum guarantees.

In real life advanced knowledge especially professional knowledge (Schön, 1984, 1990) is irreducible complex, uncertain, instable, unique and governed by value conflicts, which are not solved by reason but by power. Without going into details (see more elaborated: Baumgartner, 1993) the characteristics of real life professional knowledge assumes that we live in an inherently turbulent environment with indeterminate problematic situations, which most of the time "are not in the book".

This supposition generates a paradox: How can we teach problems nobody ever has confronted let alone solved? How can teachers teach so that students become better teachers than the ones they learned from?

In a wonderful short science fiction story Isaac Asimov (1986) reflects on this apparent paradoxical situation: Children brought up in a futuristic society have to undergo a special test where it is determined which profession they are going to practice. All the knowledge of former generations is transferred directly in their brains by a special tape during the so-called Reading Day. Only the protagonist of the story is not treated by tapes but moved to a secret but wonderful and lazy environment where he is supposed to go around, to read, to talk to other persons who weren't treated by the tape either. Shame and pain characterized the feeling of the protagonist who was seemingly treated so differently from all his friends and who was not educated ("tapped") for a special profession. What surprise as he learned that his apparently non-education was a special education for a special profession: He was supposed to become a tape builder, a profession responsible for new knowledge programmed into the tapes to guarantee the advancement of this futuristic society.

Sure, this analogy must not be taken literally: If we want to teach students to step onto the shoulders of teachers, to invent new things and to produce and generate new knowledge we have to provide a special learning environment. In this respect the analogy still holds. But instead of a lazy environment it has to be a challenging environment, which is sufficiently complex, uncertain, instable and unique so that old traditional knowledge or solutions do not work anymore.

In a certain way this model is not a teaching model any more. There is no complete control of the learning situations by the teacher anymore. Teachers and learners alike have to immerse into a situation where the outcome is not predetermined. They both have to master situations at hand and the differences between teachers and learners maybe are only more experiences and more meta-knowledge on how to reflect on complex situations (e.g. how to design local experiments) on the teachers' side.

Learning/Teaching III has strong links to constructivism. Constructivism refuses a so-called "objective" description (representation) or explanation of reality. Reality is considered as an interactive conception where observer and observation object are mutually and structurally linked. Even pure observation itself is a kind of activity, which influences the observed thing. In this aspect reality is in relation to the observer as we can see not only in social science (e.g. to observe a human changes his/her behaviour) but also in physical science (e.g. relativity and quantum theory show that the outcome of some experiments depends on the mode of observation).

In order to avoid misunderstandings it is important to see that constructivism does not neglect the external world, does not support the philosophical theory of solipsism. Constructivism only says that there is no reality "out there" which can be perceived without a subject, the human mind. There is no "objective" god's eye, independent from a perceiving human mind. Neurophysiological studies show that our sensory organs do not just transfer the inputs form the outer world to our mind, but already come up with structures and interpretations during the processing stages. We see not colours and shapes but gestalt.

From a constructivist point of view learning is considered as an active process in which people construct their knowledge by relating it to their previous experiences in complex and real situations in life. In their practical lives people are confronted with unique, unpredictable situations the problems of which are not yet obvious. Therefore, in contrast to cognitivism, the solving of already existing problems is not the main priority, but the independent generating of the problem. These must be searched for in confusing, insecure, unpredictable and partly chaotic situations.

As in Learning/Teaching II where teachers try to help individual learners in their learning process there is a individual component in Learning/Teaching III as well. Students are constructing their knowledge by relating it to their previous experiences and lives. In that respect it is by no means Objective Knowledge in the Popperian sense (Popper, 1972) but Personal Knowledge as Michael Polanyi has coined it (Polanyi, 1974).

Learning/Teaching III requires a special two-way communication structure very different from Learning/Teaching II. In model I the communication is preset and controlled by the teacher whereas in model II and III the communication is on equal terms. But there is a crucial difference in Learning/Teaching II and III: While the communication in model II is predominantly linguistically structured in model III there is also a non-linguistic representation. This could be either for example bodily performance as in dance, sports and other action types or a special notation system as it is usable e.g. in music, chess or – in mathematics. Either the problem thing is too complex, too multi-faceted to express it in the serial structured language or the action process itself has inner qualities (body feelings, holistic indivisible characteristics), which prevent an adequate verbal representation in natural language.

In Learning/Teaching III both teacher and learner are not only mentally but also bodily structurally coupled e.g. they function as intertwined systems. They learn from each other at the same time as they teach each other. The teacher can fail in mastering the situation and has his or her authority only by virtue of the greater experience and the trust the learner has to the teacher's guidance. The teacher takes the role of a "coach" or panel member in a discussion and thus loses his seemingly infallibility. A football trainer, for example, may not always successfully kick goals, or even be one of the best players of the team. Accordingly a teacher is confronted with the criticism of the reality, of practical situations. Teachers make use of their teaching functions by their experience and capabilities of assisting others dealing with complex situations.

From a curricular structured educational situation it does not make always sense to tackle complete unexplored territory. Instead of trying to discover a proof for a mathematical relationship which belongs to cutting-edge research and not to learning teachers would invite students trying to **re**-discover laws through guided and systematic explorations.

Example 5: The student gets some solved derivations and is invited to abstract (to reinvent) the rule of derivation for composite functions. In order to proof their hypothesis the students get additional tasks where they can apply their solution.

Planning and producing various outcomes with slider and different parameters and evaluating, checking, and critiquing the outcomes in different ways are the most important cognitive task under the constructivist model. Self-determined explorations are especially suited for learning with technology support. Especially in Mathematic there are marvellous possibilities with the new generation of CAS-tools (Computer Algebra Systems) like Geogebra (http://www.geogebra.org), Sage (http://www.sagemath.org/) or graphic calculators with built-in CAS like TI-Nspire (http://education.ti.com/en/us/products/calculators/graphing-calculators/ti-nspire-cx-cas-handheld). Many examples of guided explorations can be found in modern new mathematic text books like (Heugl, 2014) and are also supported by online material (http://www.veritas.at/online-angebot/mathe-mit-technologie/). Adequate tools like computer-based simulation models, graphical representations help to analyse, evaluate and find algorithm or mathematical models for authentic situation with complex and unadjusted datasets.

2 Scaffolding the individual development of competences

2.1 Learning and teaching strategies

The following graphic summarises and compares the three different prototypes of education. The rationale for the representation of all three modes in one graphic is the fact that there is no unique best model but all three educational modes have their advantages and valuable roles for the individual learning career of pupils and students.

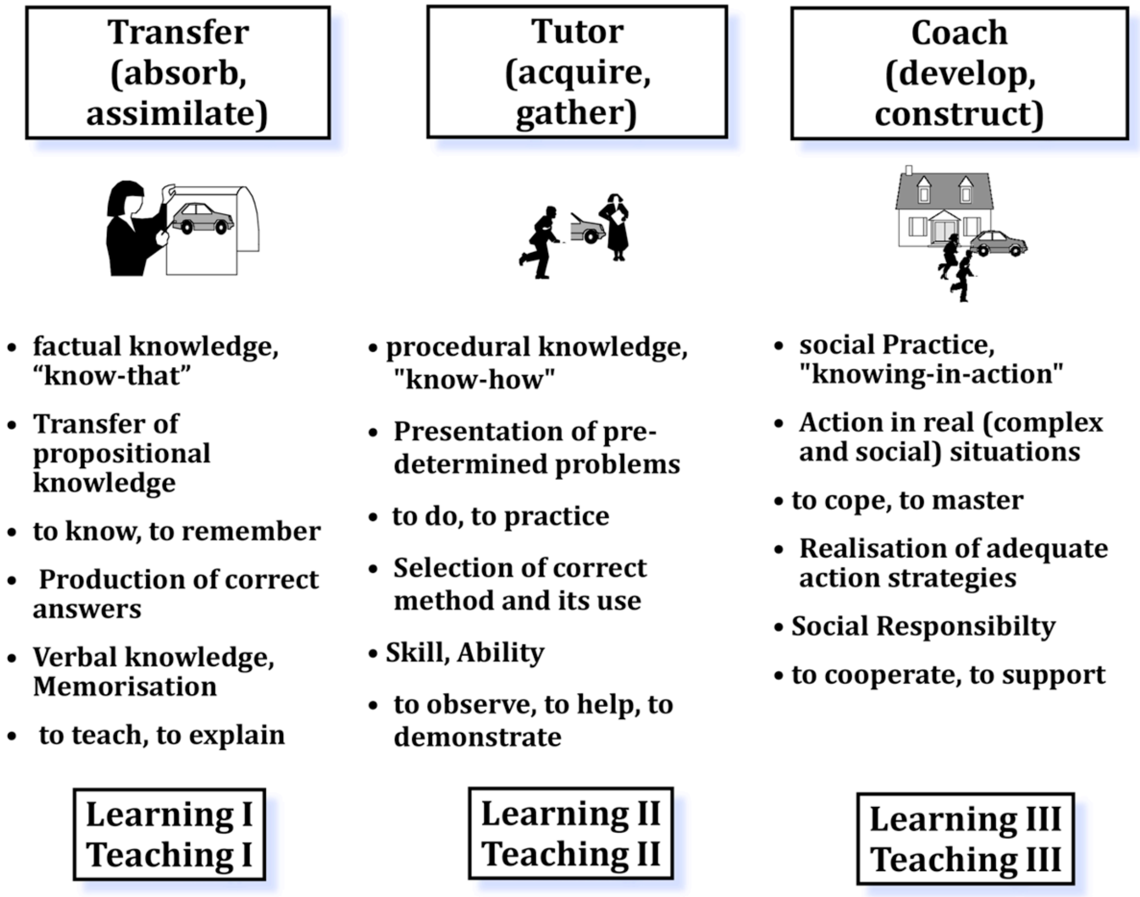

Fig. 1: Learning/Teaching modes

I make this assertion in a time where educationalists are still very fascinated with the constructivist model. It is a kind of fashion to talk about situated learning and authentic learning environments (Lave & Wenger, 1991; Choi & Hannafin, 1995). Indeed the constructivist approach supports self-determined actions and therefore the development of self-determined personalities (Deci & Ryan, 2012, 2002; Ryan & Deci, 2000). But be aware that naturalistic environments – which are often turbulent and chaotic – require already a high level of competences for successful learning experiences. Otherwise the student will be overwhelmed by the sheer dynamic of the environment and getting constructive (positive or negative) feedback that can be turned into learning experiences.

When I learned scuba diving in Mexico I was happy that the instructor was not a 100 % constructivist. Instead of immediately diving 10 meter under sea level we started with explanations (Learning/Teaching I) and continued with practices in the pool using our full equipment (Learning/Teaching II). Only after all the necessary skills were built up we went into the sea.

I have presented the three different types of learning/teaching modes in an abstract way so that they are neutral concerning the subject domain. Each teaching model can not only be used for humanities like sociology but also for technical sciences like electrical engineering or – as in our case here – for Mathematics education. Clearly enough the problems are in each domain different and maybe their construction presents different levels of difficulty for both, the teachers and the learners. The adequate application of these models is guided by the suitable teaching methodology (subject didactics) which itself has to be fitted for the subject in question.

2.2 Technology supported learning

2.2.1 Computer software

On the other hand all teaching models are also neutral against the media they use. So we can imagine computer software for all three models ranging from programmed instruction (Learning/Teaching I) to tutorial software (Learning/Teaching II) to complex simulations and/or so-called micro worlds (Learning/Teaching III).

Keep in mind, that so-called interactive software not necessarily belongs to Learning/Teaching II or III (Baumgartner & Payr, 1999). Many times the interaction just gives the necessary commands for applying different software functions (= functional interaction). The crucial point is not interactivity itself, but if the interaction has an educational (didactical) background (= educational interaction). If the automated feedback is only true/wrong then the software only supports model I. Only if the educational interactions are watched and analysed either by the human teacher or the programme in order to give concrete feedback to the student to improve his or her performance then model II or III can be supported as well.

This is also true for the Internet. Sometimes it is said that the inherent nature of the Internet brings the real world into the classrooms and therefore it clearly advocates model Learning/Teaching III. But note: The Internet can also be used for Teaching I (transmitting PDF-Files or presenting web pages without hyperlinks or a narrow set of predefined sets of hyperlinks).

2.2.2 Websites

When we are talking about the Internet we have to distinguish two different types of support. On the one hand we have a huge – and still growing – amount of websites with material. For the preparation of this article I have prepared a (non-systematic) collection of links on my weblog: http://peter.baumgartner.name/goodies/linklisten/links-zu-mathematik/. There is a form at the end of the link list. Feel free to add your suggestions to this collection.

Most of these websites offer teaching material for Learning I (explaining concepts and procedures) on all levels. Outstanding in this respect are the Mathematic portals of Wikipedia, which exist with slightly different material in many languages. But there is also a growing number of sites out there providing exercises and training material (Learning II).

Pages supporting constructivist modes of learning are rare but exist nonetheless. A good example for Learning III is "Project Euler" (http://projecteuler.net) consisting of series of challenging mathematical and computer programming problems. It combines the self-determined construction of mathematic and programming skills in a playful manner. It does not offer specific guidance to the problems but rather advocate self-regulated research with book or with the Internet. Only when one has entered the right answer, then the website opens up a discussion thread where one can see how other members have solved the problem, discuss methods, and share insights.

2.2.3 Learning Management Systems (LMS)

A completely different support modus is provided by Learning Management Systems (LMS). One well-known example is the Open Source platform moodle (https://moodle.org). These types of tools do not provide substantial assistance in the first place but support learning organisation, and the administration of learning environments for dislocated learners.

A new LMS course has no content but provides tools to administer content, students, tasks, exercises and assessments. Modern platforms have rudimentary tools for content creation as well (editors, discussion forum, wikis) but specialised content has to be created outside of the platform and then uploaded. Sometimes external software is in the platform so well integrated that it appears as a function of the platform itself.

It is important to see that the above outlined learning and/or teaching modes have their equivalent in learning management systems as well. All three learning modes can be either organised completely as eLearning experiences (= Distance Education) or combined/mixed with face-to-face phases (= Blended Learning).

The higher the chosen learning/teaching modus is, the higher are the necessary platform skills for teachers and students. This law is important and can hinder efficient learning because in addition to the subject in question one has to learn the interface and function of the platform as well. Therefore it is important to consider the educational surplus value of learning management systems. The future long-term development indicates that usage of learning platforms will be a basic competence for learning supported by growing computer skills and improved interface design.

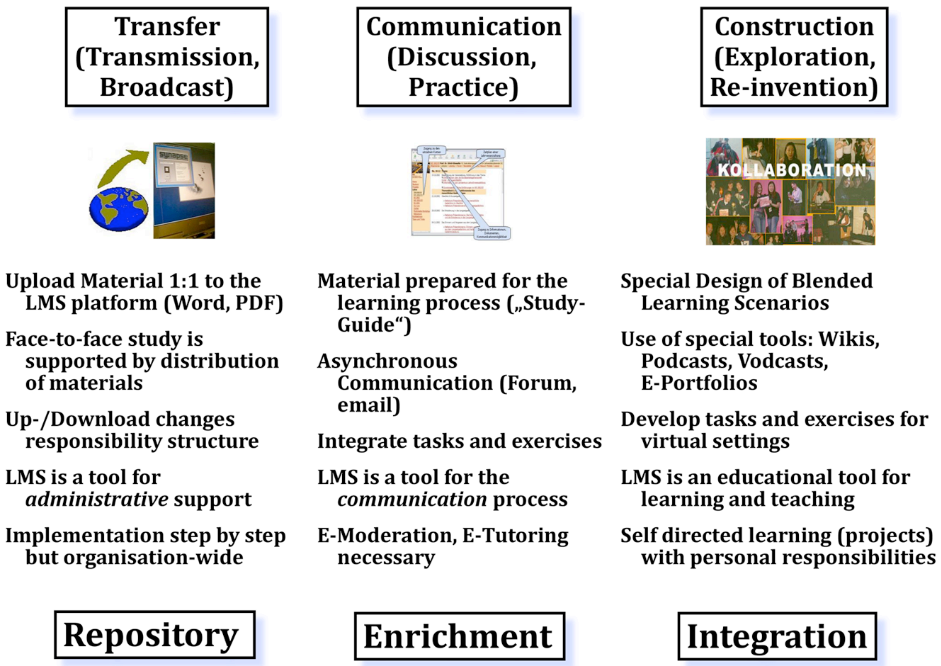

Fig. 2: Learning/Teaching modes with Learning Management Systems (LMS)

2.3 *Taxonomy for Learning, Teaching and Assessing*

In describing the learning/teaching modes I have used vocabulary drawn from the taxonomy for Learning, Teaching and Assessing (L. W. Anderson, 2002; L. W. Anderson & Krathwohl, 2001; L. W. Anderson, Sosniak, Bloom, & National Society for the Study of Education, 1994). Their work is a modern revision of the famous taxonomy of educational objectives by Bloom (1956). The taxonomy essentially consists of a two dimensional table formed by four types of knowledge (factual, conceptual, procedural and metacognitive knowledge) and six cognitive categories (remember, understand, apply, analyse, evaluate, create). These six categories not only indicate cognitive processes with different degree on complexity but the higher processes also include the lower ones. For instance: The cognitive process "apply" imply to understand the situation and to remember the adequate procedure.

On the basis of these inclusive hierarchically structured cognitive processes one can interpret the different categories as a learning sequence from simple to complex student and teacher have to follow. Figure 3 shows that the three models fit quite well into the taxonomy and can also be interpreted as a learning/teaching strategy: Starting with transfer of definitions and concepts to practice their application and analysing their consequences to explore and re-invent their relationships and rules of interaction. Under this perspective the different learning/teaching models can be interpreted as different methods to provide optimal scaffolding for the individual learning career of a student. This is a "plea for educational variety" – so the English translation of the subtitle of my book on teaching methods (Baumgartner, 2011) – as there is no one best method. Or as Helmut Heugl said: The best method is the variety of methods (quoted from Bruder, 2014).

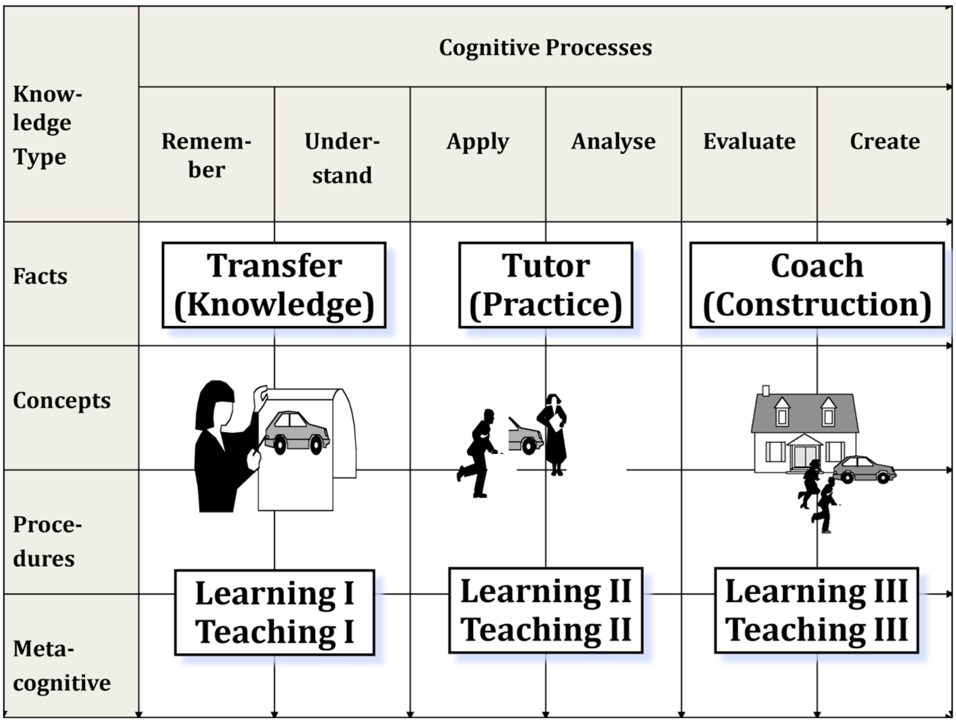

Fig. 3: Relationship of the Taxonomy and Learning/Teaching modes

2.4 The Competence Helix

It is important to understand that the three different modes do not follow each other in a pure linear fashion. Better suited is the model of a spiral or helix where at the end of the three learning modes (or six cognitive processes) the process starts again but this time from a higher level.

1. **Learning/Teaching I**: At the starting point the beginner needs some abstracted knowledge to provide the theoretical foundations and to get some signposts, road markings and orientation points. This kind of factual knowledge is static and has no value by itself in a real and complex situation. It serves just as a shortcut to prevent to fall into traps and to help to organise his or her experiences without too many failures.
2. **Learning/Teaching II**: In this section of the individual learning career the student applies the abstract knowledge and makes his or her own experiences. In order to limit the action and reflection possibilities the learner interacts with a somewhat restricted, artificial environment, which is reduced of complexity and easy to control by the teacher. To provide feedback this environment is designed in a way that includes some devices where students can deposit their interim product and teachers can inspect it. It is a kind of Zen art to construct this observation points in a way that they fit naturally into the learning environment and do not disturb or alter the learning process.
3. **Learning/Teaching III**: Teacher and learner work together to master problems. This model includes the generation or invention of the problem. The environment is constructed in a way that it represents at least in certain aspects reality or it is reality constrained by certain variables. There is a two-way communication on equal terms using either linguistic representations or other adequate kinds of languages.
4. **Learning/Teaching I+**: After the knowledge loop is completed the learner starts the loop from scratch but on a higher level or in another domain. Instead of just acting learners are revising their actions and experiences and try to improve or debug their performances.

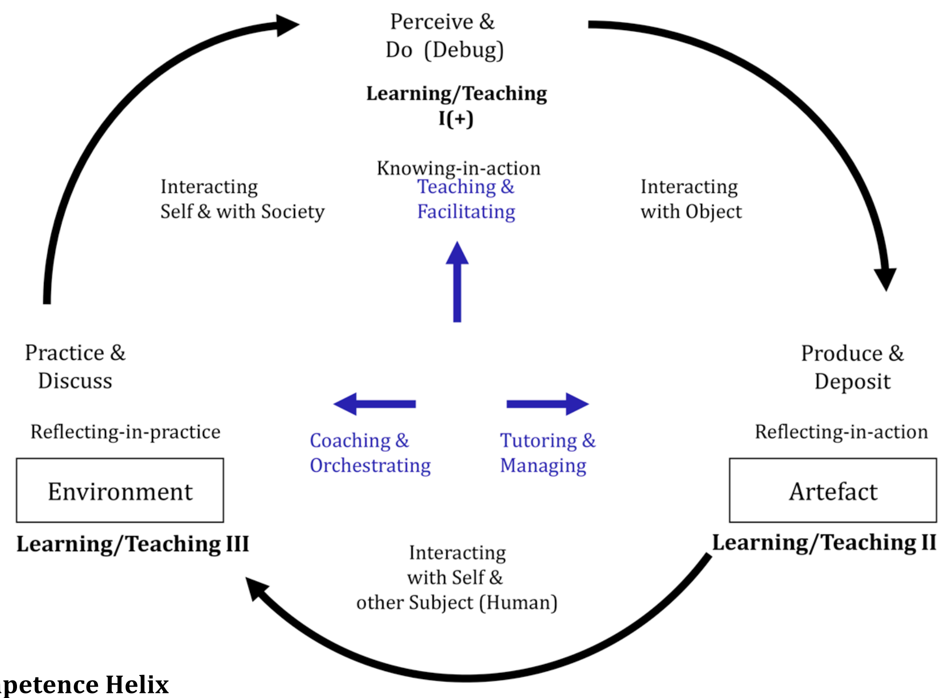

Fig. 4: The Competence Helix

2.5 Design of Blended Learning Arrangements

For Blended Learning scenarios one not only has to plan the virtual arrangements but has to (re-)think the complete educational design. E-Learning phases and contact or face-to-face time have to support each other. It does not make sense to deliver a lecture with content already discussed on the platform. The advantages and disadvantages of online and presence phases has to be taken into account in the overall design of the module/course.

To exploit the advantage of self-determined learning and to give student the chance to follow their own learning pace it does not make sense when most of the online-learning is structured with synchronous arrangements like webinars. In asynchronous settings one has to distinguish between learning time and physical time. At my department we plan the size of modules with 3 credit points, which is in Austria equivalent 75 learning hours (1 ECTS = 25 h learning time).

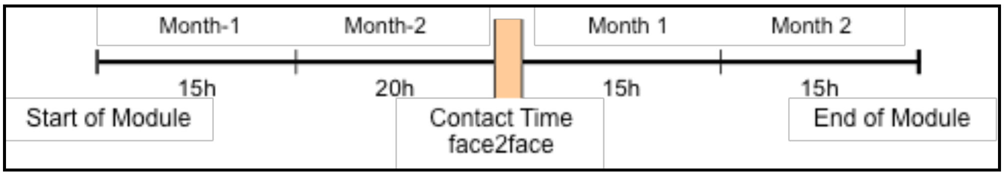

Fig. 5: A prototypical Blended Learning module (AT: 3 ECTS = 75 h learning time)

It is important to see that different learning/teaching modes require different kind of organisational structures. The higher the cognitive processes, which are necessary for the targeted learning outcome, the higher must be the necessary tutorial support for the corresponding online phases.

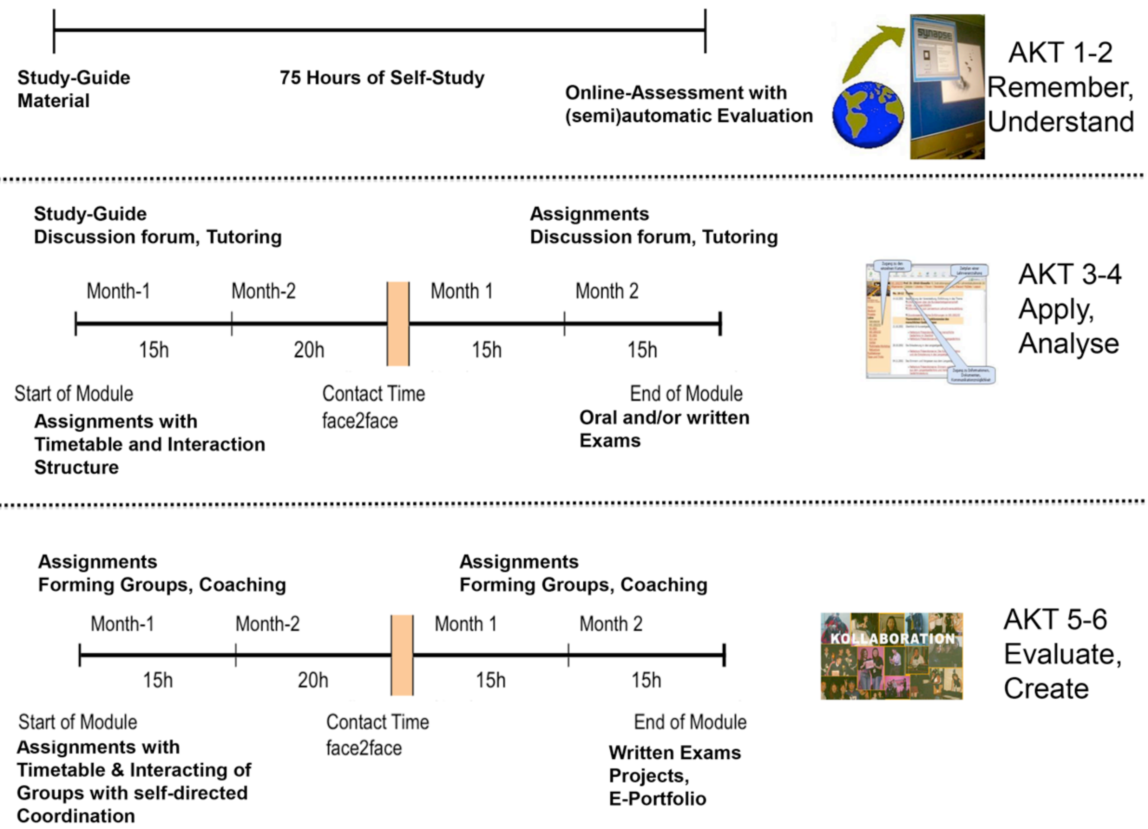

Fig. 6: Modules with different learning outcomes, categorised by the Anderson-Krathwohl Taxonomy (AKT). Learning/teaching modes, tutorial support and assessments have to be aligned by the targeted learning outcomes.

The amount of contact time to online time (the mixing ratio of the Blended Learning arrangement) has to be designed in dependence of subject, learning/teaching mode und learning outcomes. Some subjects are less suitable for eLearning than others and need therefore more contact time. For instance our course of eEducation has a higher ratio of online time as our course on educational leadership. Also the place and distribution of the contact time has to be planned and is also dependent from subject and learning goals.

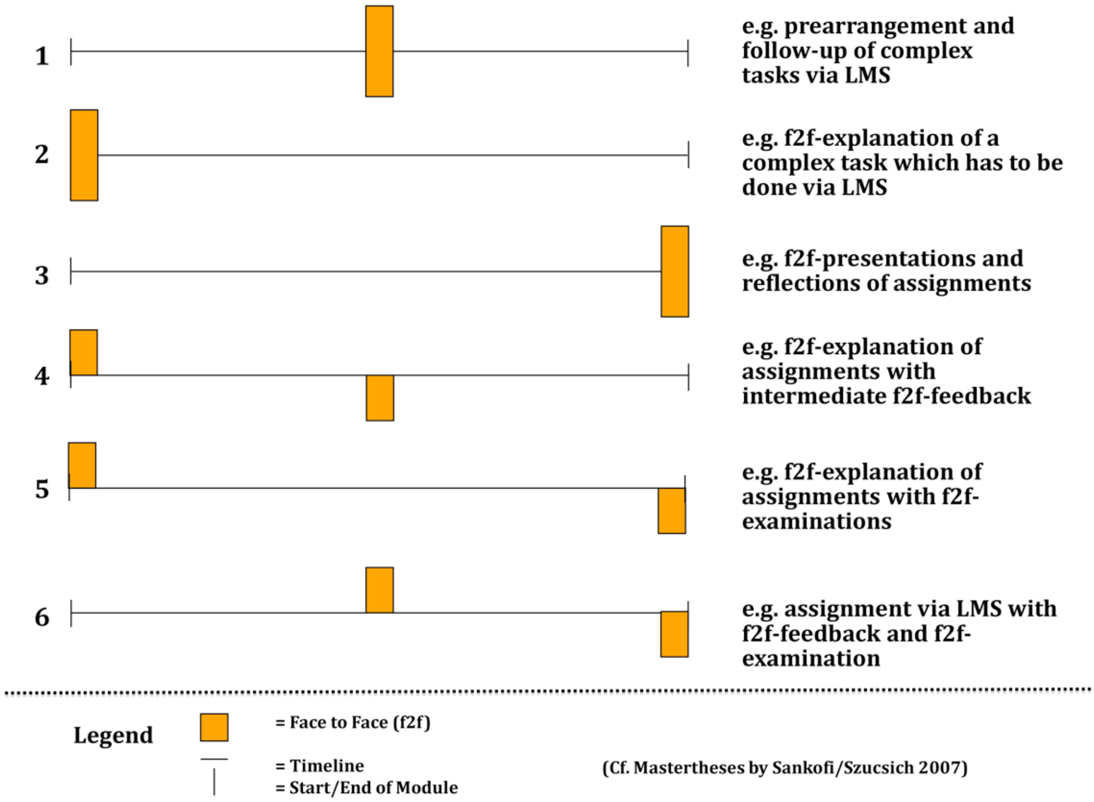

Fig. 7: Different distribution of contact time (face to face time) with different learning/teaching modes and different learning goals (Sankofi & Szucsich, 2007).

3 Case Studies

After these general considerations I want to demonstrate the power of the suggested perspective and analyse the potential of Internet application for mathematic. I will choose two very different use cases: On the one hand I will discuss new possibilities related with mobile learning and on the other hand I will report on the Polymath research project. Both cases span a great distance of learning modes: Mobile learning is suited for individual short duration learning in turbulent environments (sitting in a café, travelling with public transport, waiting to be next in an administrative office etc.), whereas the polymath case reports on a big and long time collaboration of mathematic research.

3.1 Mobile learning

Mobile learning is traditionally associated with short time learning or Microlearning. Former typical use cases were revising short text sequences, repeating and memorizing definitions or simple procedures which all belong to the mode of Learning/Teaching I. Small interfaces uncomfortable to operate and humble computing power were the main reasons for the underlying assumptions of simple learning modes. But even though these tools are very convenient. Even if they are predominantly linked with Learning/Teaching I they can also be used rudimentarily for mode 2 as the next example demonstrates:

Example 6: MathRef is an app for quickly finding formulas across multiple disciplines. It is focused on traditional math fields such as Algebra, Geometry and Calculus. MathRef allows users to add notes to equations, to save favourite equations and to copy text from within the app to e-mails or text editor. It includes useful tools as a unit converter, quadratic solver, and triangle solver to perform common calculations.

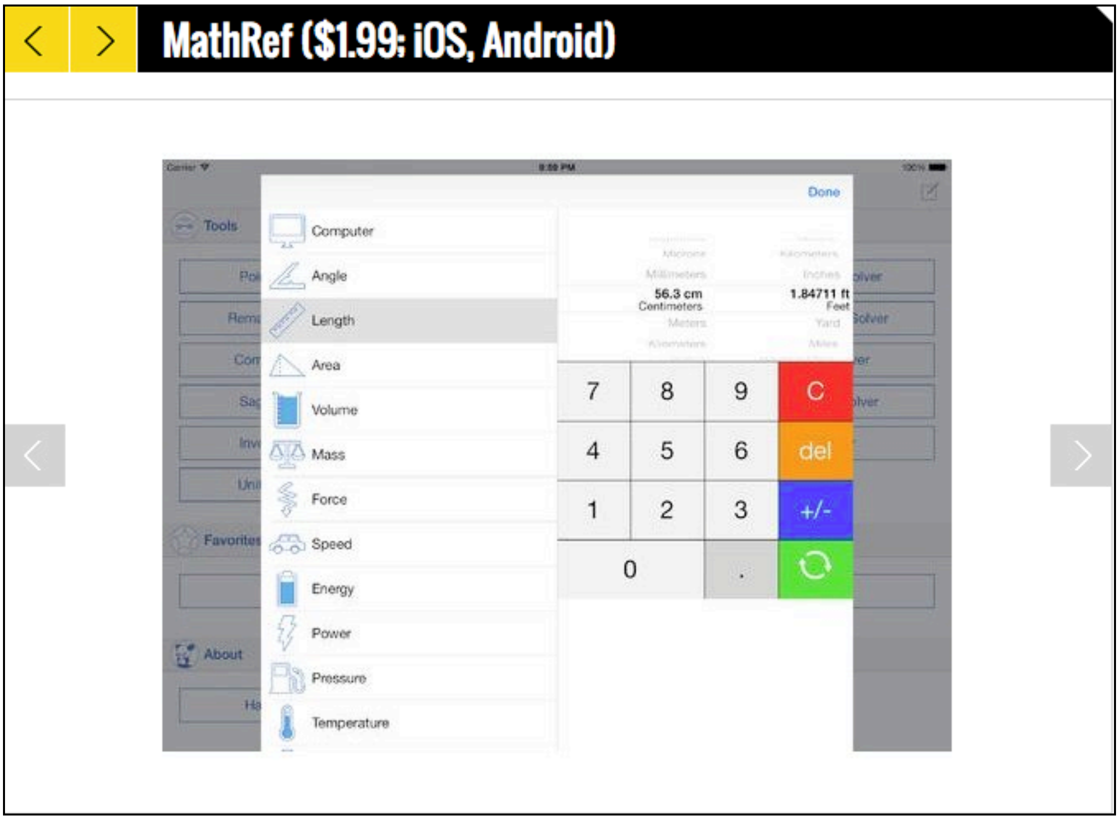

Fig. 8: MathRef – Browse over 1,400 formulas, figures, and examples and perform common calculations.

But in the meantime the situation has changed tremendously: Not only are computer chips nowadays much faster and have more memory but are also available with specialised Apps on tablet PCs supported by cloud services. We have reached a new level of computing power for small gadgets. Meanwhile there are apps around where you can perform pretty complex task on learning/teaching level II and even on level III. The next two examples give you a flavour of this new generation of apps and the possibilities behind it.

Example 7: WolframAlpha offers answers to many math-related or number-centric questions. The computational knowledge engine can compute across 29 disciplines like mathematics, statistic and data analyses, physics, chemistry, engineering, money & finance, socioeconomic data, linguistics etc. You can get formula details, graphic representations and brief explanations to help you understand how the app arrived at a given solution.

Example 8: There are different sophisticated graphic calculators available – some of them for free. A good example is the graphic calculator by MathLab: This free app provides its users with advanced operators, functions, an intuitive user interface and beautifully laid out graphs featuring slopes, roots and intersections – just to name a few. It has multiple functions on a graph, polar graphs, graphing of implicit functions, values and slopes, roots, extremes, intersections. Algebra: polynomials, polynomial equation solving, matrices, fractions, derivatives, complex numbers and more. It shows results as you type. You can use the menu to switch between different modes.

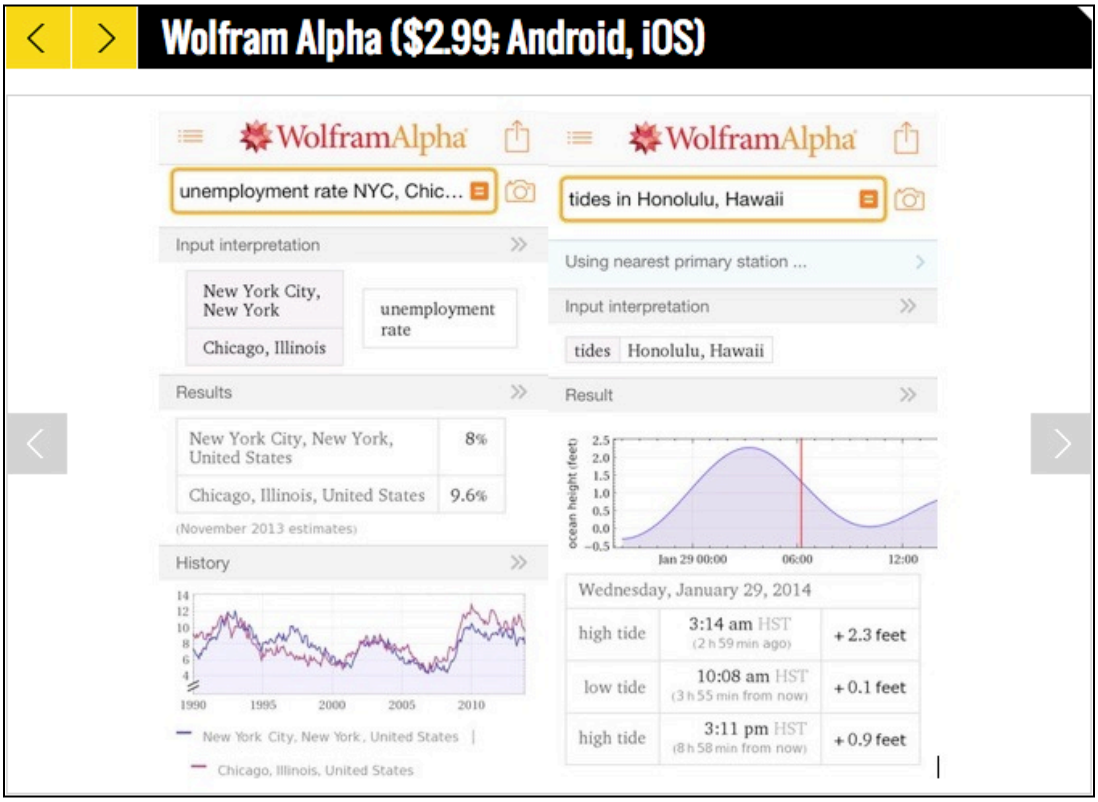

Fig. 9: Wolfram Alpha app answers to many math-related or number-centric questions.

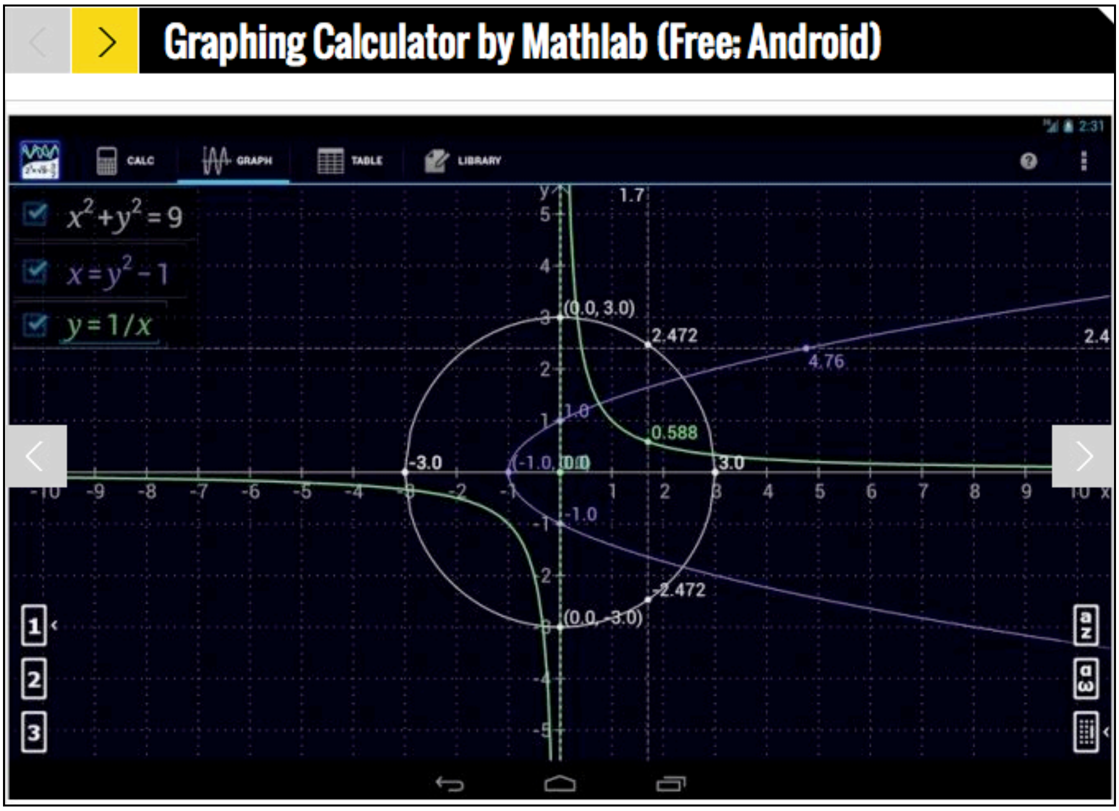

Fig. 10: Graphic calculator by MathLab provides advanced mathematical operators and functions.

But these examples only demonstrate that the small devices (smart phones and tablet PCs) are already competitive with stand-alone computers in processing power and functionality. New educational possibilities as communicating results to other user, working on a problem together etc. are still only in the tentative beginnings.

3.2 The Polymath Project and Web 2.0

When we talk about Web 2.0 we address a complete new usage of the Internet. Instead of visiting websites just as a visitor who consumes (reads the content), we want to emphasize the active participation of Internet users and focus on their contribution to the content. The Internet user becomes a "prosumer", a person who produces and consumes at the same time.

In talking about "Web 2.0" we presuppose a period, which we could call the first version of the Internet, or "Web 1.0". There is much discussion going on what could be the decisive difference in the dynamics of the Internet that allows us to speak about a complete new version. There are many new small features that alleviate the active participation of users, but in my opinion it is the different type of network connection that has changed and allows us to speak about version 2.0. In the adolescents' years of the Internet (= "Web 1.0") the main new property that converted the Internet to a radical new resource was the invention of the hyperlink. The hyperlink can be placed on every text chunk or graphic part and with just a click we could be directed immediately to another text or graphic.

Whereas Web 1.0 has predominantly connected content, Web 2.0 is connecting people. It not only connects people per se but people with the same interests, concerns and hobbies. Special software functions (so called "Social Software") observe our activities with the websites and recommend us either similar products ("Customers who bought book X have also bought book Y") or give us access to people who have demonstrated the same or similar activity pattern.

Let us put aside here in this discussion the possible misuse by advertisement, selling our activity patterns to other enterprises or other kind of penetration in our private sphere. In contrast to standard collaboration software ("groupware") where also people work together virtually Web 2.0 is not a grouping formed by organisational requirements (e.g. all members of the same compartment, all students registered into the same course etc.) but a configuration resulting from the real activities over the Internet.

Social Software works best if there is already an on-going massive interaction or collaboration on the website. The software needs a big data basis in order to get reliable and detailed activity patterns. But these patterns of activity within Web 2.0 are radically different from our used form of participation where we follow the ideal that everybody should contribute more or less the same amount to be on equal terms with other group members. In Web 2.0 the typical participation pattern is not the Gaussian normal distribution but the Pareto distribution: Few people or objects contributes the majority, the rest is provided by a "long tail" of users or objects (C. Anderson, 2006): Some examples:

- **Wikipedia**: Very few people write most of the articles and many people add sentences or correct typos. The activists are motivated by these corrections, as they know that many people are reading their content and find their contribution useful.

- **Amazon**: Only very few books are bestsellers. But even if the vast majority of books are slow sellers ("shopkeeper") their huge amount are already 30% of the sales figures.
- **MOOC**: In Massive Open Online Courses very few participants are active and take all examinations, many people are passive and just bystanders. Even if they are not very active they learn during this online event ("witness learning").

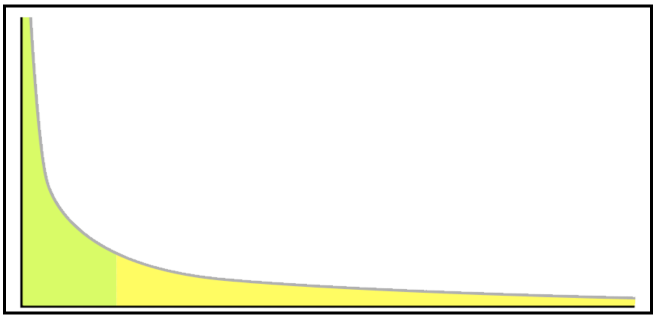

Fig. 11: Internet activities as Pareto distribution: The very long tail guarantees the success.

The question arises if collaboration over the Internet can only be done on simple tasks. The Polymath Project proofs that it can also be used for cutting-edge research collaboration (Cranshaw & Kittur, 2011). Currently there are nine official Polymath Projects. The general blog can be found under http://polymathprojects.org and there is also a wiki to ease the collaboration (Wiki: http://michaelnielsen.org/polymath1/). Until now the project has resulted
into three different scientific articles which are published under the pseudonym of D.H.J. Polymath (Polymath, 2009, 2010, 2014). Each of these projects focuses on a significant unsolved problem in mathematics. My report will focus on the first project (Polymath1: The Hales–Jewett theorem).

The Hales–Jewett theorem asserts that for every r and every k there exists n such that every r-colouring of the n-dimensional grid {1,...,k}n contains a combinatorial line.

I do not have enough competences in mathematic to appreciate the research problem. In my naïve understanding it looks for the mathematical proof of in a kind of tic-tac-toe game, which cannot end in a draw, no matter how large *n* is, no matter how many people are playing provided only that it is played on a board of sufficiently high dimensions.

Timothy Gowers introduced the call for collaboration on the Hales-Jewett theorem in his blog (http://gowers.wordpress.com/). He is a very distinguished British mathematician and received 1998 the field medal (a kind of Nobel prize for Mathematicians) for his "research connecting the fields of functional analysis and combinatorics" (http://en.wikipedia.org/wiki/Timothy_Gowers). Shortly after his announcement Terence Tao stepped into the collaboration project (http://terrytao.wordpress.com/). He is an Australian-American mathematician also well-known and famous, co-receiver of the field medal in 2006 and "the youngest person ever promoted to full professor at the age of 24 years" (http://en.wikipedia.org/wiki/Terence_Tao).

From February 1 - May 23, 2009 there were on 14 blog entries (Gowers:8 / Tao: 6) 1555 comments produced by 39 contributors. 1228 comments were on the subject itself, the rest were meta-comments about the organisation and communication structure of this voluntary, non-paid project. Most of the comments were from Gowers (285) und Tao (232), 21 collaborators provided 5 comments or less, 13 participants wrote just one comment. This distribution is no surprise as it shows the form of the Pareto curve.

If only 2 persons (Gowers and Tao) bear the brunt, can we really assume that it was a massive collaboration? A detailed analysis shows that there were important contributions by other participants than Gowers and Tao and that even the mathematicians with little experience (measured by their amount of publications) could add important ideas to the overall research project.

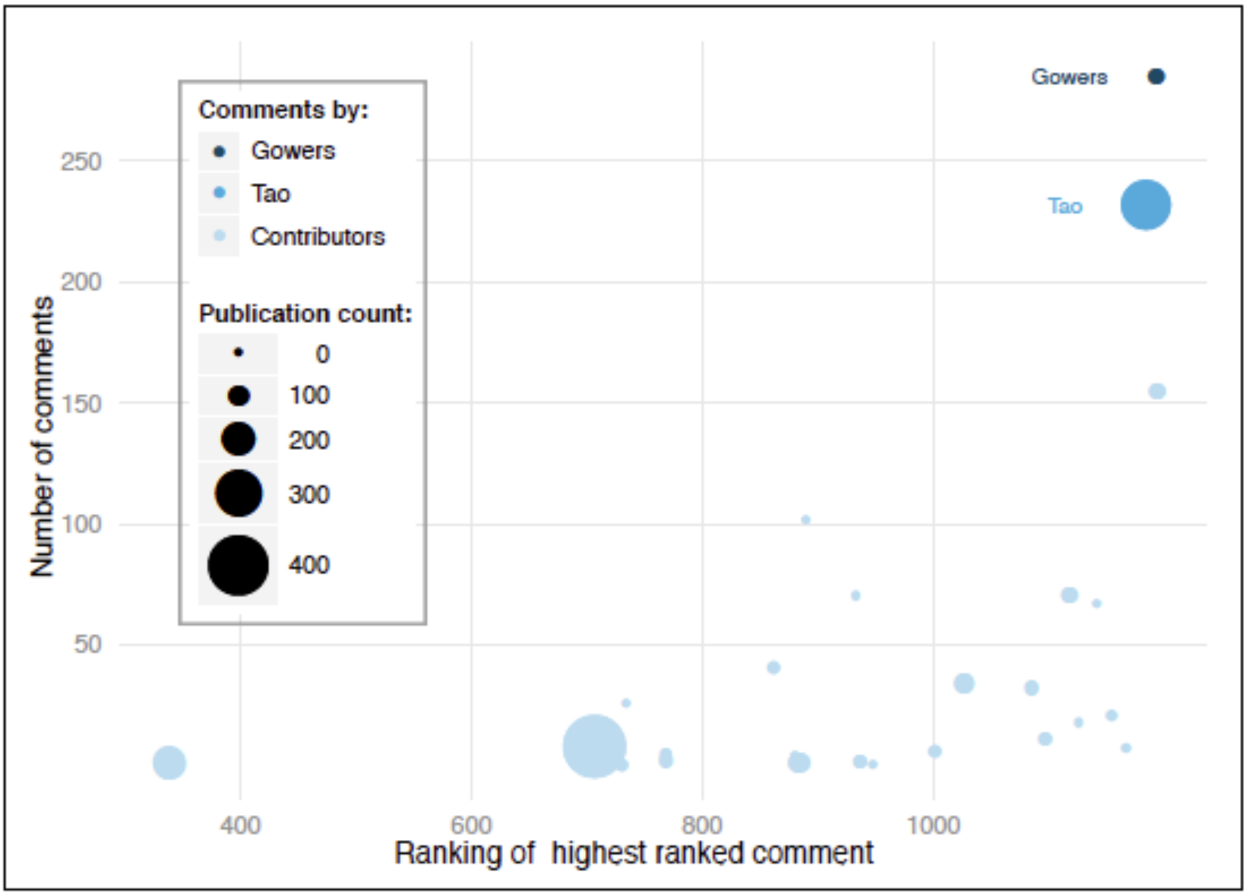

Fig. 12: The graphics show the ranking of the highest ranked comment for each user together with the total number of comments from that user. Points are sized proportional to the number of scholarly publications of that user (as given by Google Scholar) (Cranshaw & Kittur, 2011, p. 1872).

As one can see from figure 12 there are many comments from users on the right hand side of graphic, meaning that these comments were very valuable. One collaborator provided over 150 important comments and formed with Tomothy Gowers and Terence Tao the leading group. There is no relation ship between the number of publications and the value of the comments. Perhaps this is a speciality in Mathematics as even Gowers has not many publications as the relatively small perimeter of the ball on the right upper corner denotes.

From the detailed analyses by Cranshaw and Kittur (2011) we can safely say that the Internet can also be used for massive collaborative cutting-edge research.

4 Conclusion

Instead of repeating some of my arguments I will collect my main statements as a list of bullet points:

- The Internet can be used for Mathematics education in all three different learning/teaching modes.
- For every specified educational objective we have to ask: What is the educational surplus value in using eLearning/technology? Are there other (traditional) methods fitting better the intended learning outcome?
- Technology is not educationally neutral. Not every technological tool is suitable for every learning/teaching mode and learning outcome. Every educational technology, every technology supported learning environment or internet application implements a theoretical learning model – irrespective of opinions and believes of developers and teachers.
- Different learning models are not exclusive but support each other. Educational variety instead of educational monoculture!
- Design the different types of blended learning arrangements (face2face, eLearning) holistically. Align learning/teaching modes with learning outcomes and assessments procedures.
- Learning is a social enterprise and needs active participation and personal responsibility. For effective learning arrangements we need to design learning content **and** the appropriate (technologically supported) communicational structure.
- With the Internet the importance of informal learning in combination with social software and web 2.0 will increase. We will need procedure for the Acknowledgement of Prior Experiential Learning (APEL).
- Online collaboration will increase and foster "Citizen Science"

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